ELSEVIER

Available online at www.sciencedirect.com

ScienceDirect

Physics Letters B 646 (2007) 242–247

PHYSICS LETTERS B

www.elsevier.com/locate/physletb

Friedberg–Lee symmetry breaking and its prediction for θ_{13}

Shu Luo [a,b], Zhi-zhong Xing [a,b,*]

[a] CCAST (World Laboratory), PO Box 8730, Beijing 100080, China

[b] Institute of High Energy Physics, Chinese Academy of Sciences, PO Box 918, Beijing 100049, China

Received 27 December 2006; received in revised form 16 January 2007; accepted 28 January 2007

Available online 3 February 2007

Editor: T. Yanagida

Abstract

We consider an effective Majorana neutrino mass operator with the Friedberg–Lee symmetry; i.e., it is invariant under the transformation $\nu_\alpha \to \nu_\alpha + z$ (for $\alpha = e, \mu, \tau$) with z being a space–time independent constant element of the Grassmann algebra. We show that this new flavor symmetry can be broken in such a nontrivial way that the lightest neutrino remains massless but an experimentally-favored neutrino mixing pattern is achievable. In particular, we get a novel prediction for the unknown neutrino mixing angle θ_{13} in terms of two known angles: $\sin\theta_{13} = \tan\theta_{12}|(1-\tan\theta_{23})/(1+\tan\theta_{23})|$. The model can simply be generalized to accommodate CP violation and be combined with the seesaw mechanism.

PACS: 14.60.Lm; 14.60.Pq; 95.85.Ry

Recent solar [1], atmospheric [2], reactor [3] and accelerator [4] neutrino experiments have convincingly verified the hypothesis of neutrino oscillations. The latter can naturally happen if neutrinos are slightly massive and lepton flavors are not conserved. The mixing of three lepton families is described by a 3×3 unitary matrix V, whose nine elements are usually parameterized in terms of three rotation angles $(\theta_{12}, \theta_{23}, \theta_{13})$ and three CP-violating phases (δ, ρ, σ) [5]:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

with $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ (for $ij = 12, 23$ and 13). A global analysis of current neutrino oscillation data yields $30^\circ \leqslant \theta_{12} \leqslant 38^\circ$, $36^\circ \leqslant \theta_{23} \leqslant 54^\circ$ and $0^\circ \leqslant \theta_{13} < 10^\circ$ at the 99% confidence level [6], but three phases of V remain entirely unconstrained. While the absolute mass scale of three neutrinos is not yet fixed, their two mass-squared differences have already been determined to a quite good degree of accuracy [6]: $\Delta m^2_{21} \equiv m^2_2 - m^2_1 = (7.2 \cdots 8.9) \times 10^{-5}\ \mathrm{eV}^2$ and $\Delta m^2_{32} \equiv m^2_3 - m^2_2 = \pm(2.1 \cdots 3.1) \times 10^{-3}\ \mathrm{eV}^2$ at the 99% confidence level. The on-going and forthcoming neutrino oscillation experiments aim to measure the sign of Δm^2_{32}, the magnitude of θ_{13} and even the CP-violating phase δ.

How to understand the smallness of θ_{13} and the largeness of θ_{12} and θ_{23} is a real challenge. So far many neutrino mass models have been proposed [7]. Some of them follow such a guiding principle: there exists an underlying flavor symmetry in the lepton sector and its spontaneous or explicit breaking gives rise to the observed pattern of V. This is certainly a reasonable starting point for model building, and it might even shed light on the true flavor structures of leptons and quarks.

The purpose of this Letter is just to follow the above-mentioned guideline to explore a simple and testable correlation between the neutrino mass spectrum and the neutrino mixing pattern. In the basis where the charged-lepton mass matrix is diagonal, we

* Corresponding author at: Institute of High Energy Physics, Chinese Academy of Sciences, PO Box 918, Beijing 100049, China.
E-mail addresses: luoshu@mail.ihep.ac.cn (S. Luo), xingzz@mail.ihep.ac.cn (Z.-z. Xing).

0370-2693
doi:10.1016/j.physletb.2007.01.040

hypothesize that the effective Majorana neutrino mass operator is of the form

$$\mathcal{L}_{\rm mass} = \frac{1}{2}\big[a(\overline{\nu_{\tau\rm L}} - \overline{\nu_{\mu\rm L}})\big(\nu^{\rm c}_{\tau\rm L} - \nu^{\rm c}_{\mu\rm L}\big) + b(\overline{\nu_{\mu\rm L}} - \overline{\nu_{e\rm L}})\big(\nu^{\rm c}_{\mu\rm L} - \nu^{\rm c}_{e\rm L}\big) + c(\overline{\nu_{e\rm L}} - \overline{\nu_{\tau\rm L}})\big(\nu^{\rm c}_{e\rm L} - \nu^{\rm c}_{\tau\rm L}\big)\big] + \text{h.c.}, \tag{2}$$

where a, b and c are in general complex, and $\nu^{\rm c}_{\alpha\rm L} \equiv C\overline{\nu_{\alpha\rm L}}^T$ (for $\alpha = e, \mu, \tau$). A salient feature of $\mathcal{L}_{\rm mass}$ is its translational symmetry; i.e., $\mathcal{L}_{\rm mass}$ is invariant under the transformation $\nu_\alpha \to \nu_\alpha + z$ with z being a space–time independent constant element of the Grassmann algebra. Note that this new kind of flavor symmetry was first introduced by Friedberg and Lee in Ref. [8] to describe the Dirac neutrino mass operator. Here we apply the same symmetry to the case of Majorana neutrinos and then reveal a completely new way to break it. Corresponding to Eq. (2), the Majorana neutrino mass matrix M_ν reads

$$M_\nu = \begin{pmatrix} b+c & -b & -c \\ -b & a+b & -a \\ -c & -a & a+c \end{pmatrix}. \tag{3}$$

The diagonalization of M_ν is straightforward: $V^\dagger M_\nu V^* = \bar{M}_\nu$, where V is just the neutrino mixing matrix, and $\bar{M}_\nu = \text{Diag}\{m_1, m_2, m_3\}$ with m_i (for $i = 1, 2, 3$) being the neutrino masses. From Eq. (3) together with the parametrization of V in Eq. (1), it is easy to verify

$$\text{Det}(M_\nu) = \text{Det}\big(V\bar{M}_\nu V^T\big) = \text{Det}(\bar{M}_\nu)\big[\text{Det}(V)\big]^2 = m_1 m_2 m_3 e^{2i(\rho+\sigma)} = 0. \tag{4}$$

This result, which is an immediate consequence of the Friedberg–Lee (FL) symmetry in $\mathcal{L}_{\rm mass}$, implies that one of the three neutrinos must be massless. In the FL model [8] such a flavor symmetry is broken by an extra term of the form $m_0(\bar{\nu}_e\nu_e + \bar{\nu}_\mu\nu_\mu + \bar{\nu}_\tau\nu_\tau)$ added into the Dirac neutrino mass operator, hence all the three neutrinos are massive.

One may wonder whether it is possible to break the FL symmetry in $\mathcal{L}_{\rm mass}$ but keep $m_1 = 0$ or $m_3 = 0$ unchanged.[1] We find that the simplest way to make this possibility realizable is to transform one of the neutrino fields ν_α into $\kappa^*\nu_\alpha$ with $\kappa \neq 1$. Given $\nu_e \to \kappa^*\nu_e$ for example, the resultant Majorana neutrino mass operator takes the form

$$\mathcal{L}'_{\rm mass} = \frac{1}{2}\big[a(\overline{\nu_{\tau\rm L}} - \overline{\nu_{\mu\rm L}})\big(\nu^{\rm c}_{\tau\rm L} - \nu^{\rm c}_{\mu\rm L}\big) + b(\overline{\nu_{\mu\rm L}} - \kappa\overline{\nu_{e\rm L}})\big(\nu^{\rm c}_{\mu\rm L} - \kappa\nu^{\rm c}_{e\rm L}\big) + c(\kappa\overline{\nu_{e\rm L}} - \overline{\nu_{\tau\rm L}})\big(\kappa\nu^{\rm c}_{e\rm L} - \nu^{\rm c}_{\tau\rm L}\big)\big] + \text{h.c.} \tag{5}$$

Accordingly, the neutrino mass matrix is given by

$$M'_\nu = \begin{pmatrix} \kappa^2(b+c) & -\kappa b & -\kappa c \\ -\kappa b & a+b & -a \\ -\kappa c & -a & a+c \end{pmatrix}. \tag{6}$$

We are then left with $\text{Det}(M'_\nu) = \kappa^2\text{Det}(M_\nu) = 0$, which is independent of the magnitude and phase of κ. Thus we obtain either $m_1 = 0$ or $m_3 = 0$ from M'_ν. Our next step is to show that a generic bi-large neutrino mixing pattern, which is compatible very well with current experimental data, can be derived from M'_ν.

Let us focus on the $m_1 = 0$ case,[2] in which M'_ν is diagonalized by the transformation $V^\dagger M'_\nu V^* = \bar{M}_\nu$ with $\bar{M}_\nu = \text{Diag}\{0, m_2, m_3\}$. As the best-fit values of the atmospheric and CHOOZ neutrino mixing angles are $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ respectively [6], we may decompose the neutrino mixing matrix V into a product of three special unitary matrices: $V = URP$, where

$$U = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \begin{matrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{matrix} \end{pmatrix}, \qquad R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \hat{c} & \tilde{s} \\ 0 & -\tilde{s}^* & \hat{c}^* \end{pmatrix}, \tag{7}$$

and $P = \mathbf{1}e^{i\gamma}$ with the definitions $\hat{c} \equiv \cos\theta e^{i\phi}$ and $\tilde{s} \equiv \sin\theta e^{i\varphi}$. Note that $\mathbf{u}_1^*$ is a column vector associated with $m_1 = 0$ (i.e., $M'_\nu\mathbf{u}_1^* = 0$ holds). This observation, together with the unitarity of U, allows us to obtain

$$\mathbf{u}_1 = \frac{1}{\sqrt{2|\kappa|^2+1}}\begin{pmatrix} 1 \\ \kappa^* \\ \kappa^* \end{pmatrix}, \qquad \mathbf{u}_2 = \frac{\sqrt{2}}{\sqrt{2|\kappa|^2+1}}\begin{pmatrix} \kappa \\ -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}. \tag{8}$$

One can see that U is only dependent on κ, a free parameter characterizing the strength of FL symmetry breaking in $\mathcal{L}'_{\rm mass}$. Apparently, $V^\dagger M'_\nu V^* = P^\dagger R^\dagger(U^\dagger M'_\nu U^*)R^* P^*$ holds, where

$$U^\dagger M'_\nu U^* = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & (b+c)(2|\kappa|^2+1) & (c-b)\sqrt{2|\kappa|^2+1} \\ 0 & (c-b)\sqrt{2|\kappa|^2+1} & 4a+b+c \end{pmatrix}. \tag{9}$$

[1] Because of $m_2 > m_1$ obtained from the solar neutrino oscillation data [1], it makes no sense to consider the $m_2 = 0$ case.

[2] We find that the $m_3 = 0$ case is actually disfavored, if we intend to achieve $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ from M'_ν in the leading-order approximation.

If a, b and c are all complex, the phases of $U^\dagger M'_\nu U^*$ will finally be absorbed into the phases of P (γ) and R (ϕ and φ).

For simplicity, we assume that a, b and c are all real. In this case, $U^\dagger M'_\nu U^*$ is a real symmetric matrix. Hence the phases of R and P (i.e., γ, ϕ and φ) can be switched off and the rotation angle of R is determined by

$$\tan 2\theta = \frac{(b-c)\sqrt{2|\kappa|^2+1}}{(b+c)|\kappa|^2 - 2a}. \tag{10}$$

One may observe that $b=c$, which is a clear reflection of the μ–τ permutation symmetry in $\mathcal{L}'_{\rm mass}$ or M'_ν [9], simply leads to $\theta = 0$ or equivalently $\theta_{23} = \pi/4$ and $\theta_{13} = 0$. In addition, two non-vanishing neutrino masses are obtained from Eq. (9) as follows:

$$\begin{aligned} m_2 &= a + \frac{1}{2}(b+c)\left(|\kappa|^2+1\right) - \frac{1}{2}\sqrt{\left[2a-(b+c)|\kappa|^2\right]^2 + (b-c)^2\left(2|\kappa|^2+1\right)}, \\ m_3 &= a + \frac{1}{2}(b+c)\left(|\kappa|^2+1\right) + \frac{1}{2}\sqrt{\left[2a-(b+c)|\kappa|^2\right]^2 + (b-c)^2\left(2|\kappa|^2+1\right)}. \end{aligned} \tag{11}$$

Nine elements of the neutrino mixing matrix $V = URP$ can be given in terms of κ and θ:

$$V = \begin{pmatrix} \frac{1}{\sqrt{2|\kappa|^2+1}} & \frac{\sqrt{2}\kappa\cos\theta}{\sqrt{2|\kappa|^2+1}} & \frac{\sqrt{2}\kappa\sin\theta}{\sqrt{2|\kappa|^2+1}} \\ \frac{\kappa^*}{\sqrt{2|\kappa|^2+1}} & -\frac{1}{\sqrt{2}}\left(\frac{\cos\theta}{\sqrt{2|\kappa|^2+1}} + \sin\theta\right) & \frac{1}{\sqrt{2}}\left(\cos\theta - \frac{\sin\theta}{\sqrt{2|\kappa|^2+1}}\right) \\ \frac{\kappa^*}{\sqrt{2|\kappa|^2+1}} & -\frac{1}{\sqrt{2}}\left(\frac{\cos\theta}{\sqrt{2|\kappa|^2+1}} - \sin\theta\right) & -\frac{1}{\sqrt{2}}\left(\cos\theta + \frac{\sin\theta}{\sqrt{2|\kappa|^2+1}}\right) \end{pmatrix}. \tag{12}$$

After rephasing this expression of V with the transformations of charged-lepton and neutrino fields $e \to e^{i\phi_\kappa} e$, $\mu \to -\mu$, $\nu_1 \to e^{i\phi_\kappa}\nu_1$ and $\nu_3 \to -\nu_3$, where $\phi_\kappa \equiv \arg(\kappa)$, we may directly compare it with the parametrization given in Eq. (1). Then we arrive at

$$\begin{aligned} \tan\theta_{12} &= \sqrt{2}|\kappa|\cos\theta, \\ \tan\theta_{23} &= \left|\frac{\sqrt{2|\kappa|^2+1} - \tan\theta}{\sqrt{2|\kappa|^2+1} + \tan\theta}\right|, \\ \sin\theta_{13} &= \frac{\sqrt{2}|\kappa||\sin\theta|}{\sqrt{2|\kappa|^2+1}}, \end{aligned} \tag{13}$$

where θ_{12}, θ_{23} and θ_{13} are required to lie in the first quadrant, θ is close to zero due to the smallness of θ_{13} but it may be either positive (in the first quadrant) or negative (in the fourth quadrant). Furthermore, we have $\delta = 0$ (when $\theta < 0$) or $\delta = \pi$ (when $\theta > 0$) together with $\sigma = \pi$, while the CP-violating phase ρ is not well defined in the $m_1 = 0$ case. Thus we conclude that there is no CP violation in this simple neutrino mass model, although its mass operator involves a complex parameter κ.

Now that the mixing angles θ_{12} and θ_{23} are already known to a reasonable degree of accuracy, we can use them to determine the unknown mixing angle θ_{13} and the unknown magnitude of κ from Eq. (13). Indeed,

$$\sin\theta_{13} = \left|\frac{1-\tan\theta_{23}}{1+\tan\theta_{23}}\right| \tan\theta_{12}. \tag{14}$$

This interesting expression indicates that the deviation of θ_{13} from zero is closely correlated with the deviation of θ_{23} from $\pi/4$. It is a novel prediction of our model, which can easily be tested in the near future. On the other hand,

$$|\kappa| = \frac{\sin\theta_{12}}{\sqrt{\cos 2\theta_{12} + \sin 2\theta_{23}}}. \tag{15}$$

Because of $m_2 = \sqrt{\Delta m^2_{21}}$ and $m_3 = \sqrt{\Delta m^2_{21} + |\Delta m^2_{32}|}$ in the $m_1 = 0$ case, we get $m_2 \approx (8.48 \cdots 9.43) \times 10^{-3}$ eV and $m_3 \approx (4.58 \cdots 5.57) \times 10^{-2}$ eV from the 99% confidence-level ranges of Δm^2_{21} and $|\Delta m^2_{32}|$ [6]. These results, together with the experimental values of θ_{12} and θ_{23}, allow us to numerically constrain the model parameters via Eqs. (10), (11) and (13). We obtain $0.019\ \text{eV} \lesssim a \lesssim 0.026\ \text{eV}$, $0.41 \lesssim |\kappa| \lesssim 0.56$, $|\theta| < 11.4^\circ$ and the ranges of b and c shown in Fig. 1. Note that the region of $|\kappa|$ can also be achieved from Eq. (15). In particular, $|\kappa| = 1/2$ is favorable and it implies that U takes the so-called tri-bimaximal mixing pattern [10]. The numerical dependence of θ_{13} on θ_{12} and θ_{23} is illustrated in Fig. 2, from which an upper bound $\theta_{13} \leqslant 7.1^\circ$ can be extracted. Such a constraint on θ_{13} is certainly more stringent than $\theta_{13} < 10^\circ$ obtained from a global analysis of current neutrino oscillation data [6]. It will be interesting to see whether our prediction for the correlation between the unknown mixing angle θ_{13} and two known angles can survive the future measurements.

In the above discussions we have taken $\nu_e \to \kappa^*\nu_e$ to break the FL symmetry and achieve a realistic pattern of the neutrino mass matrix. One may similarly consider $\nu_\mu \to \kappa^*\nu_\mu$ or $\nu_\tau \to \kappa^*\nu_\tau$ with $\kappa \neq 1$. In either possibility it is easy to show that $m_1 = 0$ or $m_3 = 0$ holds, but the neutrino mixing pattern turns out to be disfavored by current experimental data. We find that there is no way to simultaneously obtain large θ_{23} and tiny θ_{13} in the $m_3 = 0$ case, no matter whether $\nu_\mu \to \kappa^*\nu_\mu$ or $\nu_\tau \to \kappa^*\nu_\tau$ is taken. As for the

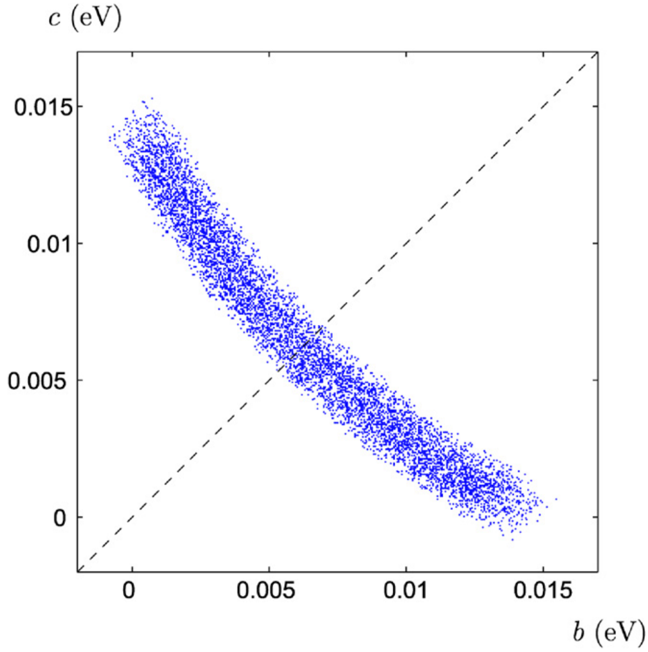

Fig. 1. The ranges of b and c constrained by current data through Eqs. (10), (11) and (13).

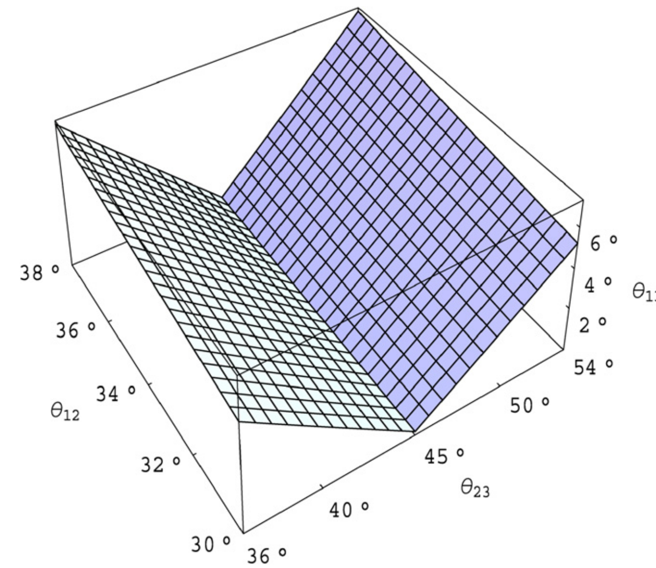

Fig. 2. The numerical dependence of θ_{13} on θ_{12} and θ_{23} as analytically predicted by Eq. (14).

$m_1 = 0$ case, it is straightforward to get the neutrino mixing matrix from Eq. (12) with the interchange of its first and second rows (when $\nu_\mu \to \kappa^* \nu_\mu$ is concerned) or its first and third rows (when $\nu_\tau \to \kappa^* \nu_\tau$ is concerned). We observe that $|\kappa| \sim 1$ is required to assure $\theta_{23} \sim \pi/4$ and $\theta_{13} \sim 0$ in the leading-order approximation, either for $\nu_\mu \to \kappa^* \nu_\mu$ or for $\nu_\tau \to \kappa^* \nu_\tau$. But $|\kappa| \sim 1$ will give rise to an excessively large value of θ_{12} (e.g., $\theta_{12} > \pi/4$), which has been ruled out by the solar neutrino oscillation data. Hence neither $\nu_\mu \to \kappa^* \nu_\mu$ nor $\nu_\tau \to \kappa^* \nu_\tau$ with $\kappa \neq 1$, which automatically breaks the μ–τ permutation symmetry, is favored to reproduce the exactly or approximately tri-bimaximal neutrino mixing pattern.

Although the discussions from Eq. (10) to Eq. (15) are based on the assumption of real a, b and c, they can easily be extended to the case of complex a, b and c in order to accommodate CP violation. For simplicity of illustration, here we assume that a remains real but $b = c^*$ is complex. One may then simplify the expression of $U^\dagger M'_\nu U^*$ in Eq. (9) by taking into account $b + c = 2\,\mathrm{Re}(b)$ and $b - c = 2i\,\mathrm{Im}(b)$. After an analogous calculation, we obtain the neutrino mixing matrix

$$V = \begin{pmatrix} \frac{1}{\sqrt{2|\kappa|^2+1}} & i\,\frac{\sqrt{2}\kappa\cos\theta}{\sqrt{2|\kappa|^2+1}} & i\,\frac{\sqrt{2}\kappa\sin\theta}{\sqrt{2|\kappa|^2+1}} \\ \frac{\kappa^*}{\sqrt{2|\kappa|^2+1}} & -\frac{1}{\sqrt{2}}\left(i\,\frac{\cos\theta}{\sqrt{2|\kappa|^2+1}} + \sin\theta\right) & \frac{1}{\sqrt{2}}\left(\cos\theta - i\,\frac{\sin\theta}{\sqrt{2|\kappa|^2+1}}\right) \\ \frac{\kappa^*}{\sqrt{2|\kappa|^2+1}} & -\frac{1}{\sqrt{2}}\left(i\,\frac{\cos\theta}{\sqrt{2|\kappa|^2+1}} - \sin\theta\right) & -\frac{1}{\sqrt{2}}\left(\cos\theta + i\,\frac{\sin\theta}{\sqrt{2|\kappa|^2+1}}\right) \end{pmatrix}, \tag{16}$$

where θ is given by $\tan 2\theta = -\mathrm{Im}(b)\sqrt{2|\kappa|^2+1}/[a + \mathrm{Re}(b)(|\kappa|^2+1)]$. Two immediate but important observations are in order:

- In this simple scenario V contains two nontrivial CP-violating phases: $\delta = -\pi/2$ (when $\theta < 0$) or $\delta = \pi/2$ (when $\theta > 0$) and $\sigma = -\pi/2$. Both of them are attributed to the purely imaginary term $b - c$. The Jarlskog invariant of CP violation [11] reads $\mathcal{J} = |\kappa|^2 |\sin 2\theta|/[2(2|\kappa|^2+1)^{3/2}]$. A numerical analysis yields $0.41 \lesssim |\kappa| \lesssim 0.57$ and $|\theta| < 19.4^\circ$. Thus we arrive at $\mathcal{J} \lesssim 0.041$. It is likely to measure $\mathcal{J} \sim \mathcal{O}(10^{-2})$ in the future long-baseline neutrino oscillation experiments.
- $\tan\theta_{23} = 1$ or $\theta_{23} = \pi/4$ can be achieved, although the neutrino mass operator $\mathcal{L}'_{\rm mass}$ does not possess the exact μ–τ symmetry. The reason is simply that $|b| = |c|$ holds in our scenario. In other words, the phase difference between b and c signifies a kind of *soft* μ–τ symmetry breaking which can keep $\theta_{23} = \pi/4$ but cause $\theta_{13} \neq 0$ [9]. Note that Eq. (16) also yields $\sin\theta_{13} = \sqrt{2}|\kappa||\sin\theta|/\sqrt{2|\kappa|^2+1}$ and $\tan\theta_{12} = \sqrt{2}|\kappa|\cos\theta$, exactly identical to the expressions given in Eq. (13).

It is worth mentioning that the present scenario has the same number of free parameters as the previous one. Taking account of current experimental data on Δm^2_{21}, Δm^2_{32}, θ_{12} and θ_{13}, we arrive at $0.026\ \mathrm{eV} \lesssim a \lesssim 0.032\ \mathrm{eV}$, $-0.010\ \mathrm{eV} \lesssim \mathrm{Re}(b) \lesssim -0.005\ \mathrm{eV}$ and $-0.013\ \mathrm{eV} \lesssim \mathrm{Im}(b) \lesssim 0.013\ \mathrm{eV}$ from a straightforward calculation.

Finally we point out that it is possible to derive the Majorana neutrino mass operator $\mathcal{L}'_{\rm mass}$ from the minimal seesaw model (MSM) [12], a canonical extension of the Standard Model with only two heavy right-handed Majorana neutrinos. The neutrino mass term in the MSM can be written as

$$-\mathcal{L}_{\rm MSM} = \frac{1}{2}\overline{(\nu_{\rm L}, N^{\rm c}_{\rm R})}\begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M^T_{\rm D} & M_{\rm R}\end{pmatrix}\begin{pmatrix}\nu^{\rm c}_{\rm L} \\ N_{\rm R}\end{pmatrix} + \text{h.c.}, \tag{17}$$

where $\nu_{\rm L}$ and $N_{\rm R}$ denote the column vectors of $(\nu_e, \nu_\mu, \nu_\tau)_{\rm L}$ and $(N_1, N_2)_{\rm R}$ fields, respectively. Provided the mass scale of $M_{\rm R}$ is considerably higher than that of $M_{\rm D}$, one may obtain the effective (left-handed) Majorana neutrino mass matrix M'_ν from Eq. (17) via the well-known seesaw mechanism [13]: $M'_\nu = M_{\rm D} M^{-1}_{\rm R} M^T_{\rm D}$. As $M_{\rm R}$ is of rank 2, $\mathrm{Det}(M'_\nu) = 0$ holds and $m_1 = 0$ (or $m_3 = 0$) is guaranteed. We find that the expression of M'_ν given in Eq. (6) can be reproduced from $M_{\rm D}$ and $M_{\rm R}$ if they take the following forms:

$$M_{\rm D} = \Lambda_{\rm D}\begin{pmatrix} \kappa & 0 \\ -1 & -1 \\ 0 & 1\end{pmatrix}, \qquad M_{\rm R} = \frac{\Lambda^2_{\rm D}}{ab+bc+ca}\begin{pmatrix} a+c & c \\ c & b+c\end{pmatrix}, \tag{18}$$

where $\Lambda_{\rm D}$ characterizes the mass scale of $M_{\rm D}$. For simplicity, we require a, b and c to be real and get the mass eigenvalues of $M_{\rm R}$

$$M_1 = \frac{a+b+2c-\sqrt{(a-b)^2+4c^2}}{2(ab+bc+ca)}\Lambda^2_{\rm D}, \qquad M_2 = \frac{a+b+2c+\sqrt{(a-b)^2+4c^2}}{2(ab+bc+ca)}\Lambda^2_{\rm D}. \tag{19}$$

Given $\Lambda_{\rm D} \sim 174$ GeV (i.e., the scale of electroweak symmetry breaking), $a \sim 0.022$ eV and $b \sim c \sim 0.006$ eV as the typical inputs, the masses of two right-handed Majorana neutrinos turn out to be $M_1 \sim 1 \times 10^{15}$ GeV and $M_2 \sim 3 \times 10^{15}$ GeV, which are quite close to the energy scale of grand unified theories $\Lambda_{\rm GUT} \sim 10^{16}$ GeV. Note that the textures of $M_{\rm D}$ and $M_{\rm R}$ taken in Eq. (18) are by no means unique, but they may serve as a good example to illustrate how the seesaw mechanism works to give rise to M'_ν or $\mathcal{L}'_{\rm mass}$ in the MSM.

To summarize, we emphasize that the FL symmetry is a new kind of flavor symmetry applicable to the building of neutrino mass models. Imposing this symmetry on the effective Majorana neutrino mass operator, we have shown that it can be broken in such a novel way that the lightest neutrino remains massless but an experimentally-favored bi-large neutrino mixing pattern is achievable. This phenomenological scenario predicts a testable relationship between the unknown neutrino mixing angle θ_{13} and the known angles θ_{12} and θ_{23} in the CP-conserving case: $\sin\theta_{13} = \tan\theta_{12}|(1-\tan\theta_{23})/(1+\tan\theta_{23})|$. Such a result is suggestive and interesting because it directly correlates the deviation of θ_{13} from zero with the deviation of θ_{23} from $\pi/4$. We have discussed a simple but instructive possibility of introducing CP violation into the Majorana neutrino mass operator, in which the soft breaking of μ–τ permutation symmetry yields $\delta = \pi/2$ (or $\delta = -\pi/2$) but keeps $\theta_{23} = \pi/4$. We have also discussed the possibility of incorporating our scenario in the MSM.

In conclusion, the FL symmetry and its breaking mechanism may have a wealth of implications in neutrino phenomenology. The physics behind this new flavor symmetry remains unclear and deserves a further study.

Acknowledgements

One of us (Z.Z.X.) likes to thank Z. Chang, X.D. Ji, M. Li, J.X. Lu and S. Zhou for very useful discussions. This work was supported in part by the National Natural Science Foundation of China.

References

[1] SNO Collaboration, Q.R. Ahmad, et al., Phys. Rev. Lett. 89 (2002) 011301.
[2] For a review, see: C.K. Jung, et al., Annu. Rev. Nucl. Part. Sci. 51 (2001) 451.
[3] KamLAND Collaboration, K. Eguchi, et al., Phys. Rev. Lett. 90 (2003) 021802.
[4] K2K Collaboration, M.H. Ahn, et al., Phys. Rev. Lett. 90 (2003) 041801.
[5] H. Fritzsch, Z.Z. Xing, Phys. Lett. B 517 (2001) 363;
See also: Particle Data Group, W.M. Yao, et al., J. Phys. G 33 (2006) 1.
[6] A. Strumia, F. Vissani, hep-ph/0606054.
[7] For recent reviews with extensive references, see: H. Fritzsch, Z.Z. Xing, Prog. Part. Nucl. Phys. 45 (2000) 1;
Z.Z. Xing, Int. J. Mod. Phys. A 19 (2004) 1;
G. Altarelli, F. Feruglio, New J. Phys. 6 (2004) 106;
R.N. Mohapatra, et al., hep-ph/0510213;
R.N. Mohapatra, A.Yu. Smirnov, hep-ph/0603118;
A. Strumia, F. Vissani, in Ref. [6].
[8] R. Friedberg, T.D. Lee, High Energy Phys. Nucl. Phys. 30 (2006) 591, hep-ph/0606071.
[9] Z.Z. Xing, H. Zhang, S. Zhou, Phys. Lett. B 641 (2006) 189;
Z.Z. Xing, S. Zhou, hep-ph/0607302.
[10] P.F. Harrison, D.H. Perkins, W.G. Scott, Phys. Lett. B 530 (2002) 167;
Z.Z. Xing, Phys. Lett. B 533 (2002) 85;
P.F. Harrison, W.G. Scott, Phys. Lett. B 535 (2002) 163;
X.G. He, A. Zee, Phys. Lett. B 560 (2003) 87.

[11] C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039;
D.D. Wu, Phys. Rev. D 33 (1986) 860.
[12] P. Frampton, S.L. Glashow, T. Yanagida, Phys. Lett. B 548 (2002) 119;
For a review of the minimal seesaw model, see: W.L. Guo, Z.Z. Xing, S. Zhou, hep-ph/0612033, Int. J. Mod. Phys. E, in press.
[13] P. Minkowski, Phys. Lett. B 67 (1977) 421;
T. Yanagida, in: O. Sawada, A. Sugamoto (Eds.), Proceedings of the Workshop on Unified Theory and the Baryon Number of the Universe, KEK, Tsukuba, 1979, p. 95;
M. Gell-Mann, P. Ramond, R. Slansky, in: P. van Nieuwenhuizen, D. Freedman (Eds.), Supergravity, North-Holland, Amsterdam, 1979, p. 315;
S.L. Glashow, in: M. Lévy, et al. (Eds.), Quarks and Leptons, Plenum, New York, 1980, p. 707;
R.N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.